

Three-dimensional lattice U(1) gauge-Higgs model at low m_H

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Received: 14 April 1997

Abstract. We study the non-compact version of the U(1) gauge-Higgs model in three dimensions for $m_H = 30$ GeV. We found that, using this formulation, rather modest lattices approach quite well the infinite volume behaviour. The phase transition is first order, as expected for this Higgs mass. The latent heat (in units of T_{cr}^4) is compatible with the predictions of the two-loop effective potential; it is an order of magnitude less than the corresponding SU(2) value. The transition temperature and $\langle \varphi^* \varphi \rangle$ in units of the critical temperature are also compatible with the perturbative results.

1 Introduction

The main reason for the study of the three-dimensional gauge-Higgs system is its relation to the full $SU(2) \times U(1)$ Standard model at finite temperature. The latter has been studied extensively in recent years in connection with the scenario of baryon violation at the electroweak scale during the evolution of the Early Universe.

It is well known that perturbation theory is not reliable for the study of such models, because of severe infrared divergencies. One promising approach has been to reduce the four-dimensional model at finite temperature to an effective model in three dimensions. This can be done if the couplings are small and the temperature is much larger than any other mass scale in the theory [1–3]. The parameters of the reduced theory are related to the ones of the original model through perturbation theory. The reduced theory has some advantages over the original one from the computational point of view [5–7]. It is super-renormalizable and yields transparent relations between the (dimensionful) continuous parameters and the lattice ones. Moreover, the number of mass scales is drastically reduced: (a) the scale T , present in four dimensions is evidently absent, (b) one may also integrate out the temporal component A_0 of the gauge field, so its mass scale gT also disappears. Thus there are two mass scales less and this reduces substantially the computer time needed to get reliable results.

The model that has already been studied along these lines [5–8] has been based on $SU(2)$ with one complex Higgs doublet. (For work on the same model in asymmetric four-dimensional lattices, see [15].) The issues studied have been the order and the characteristics (critical temperature, latent heat, surface tension, correlation lengths) of the phase transition, as well as the reliability of perturbation theory deep in the broken phase for several values of the Higgs mass. It is interesting to see how the above findings are affected by two characteristics of the study:

the compactness of the gauge group and its non-abelian nature.

The abelian Higgs model has already been studied both in three and four dimensions [14]. The compact U(1) model has been studied on the lattice in [10, 11]. An interesting aspect of the role of the abelian character of the model would be to compare its latent heat against the one of the corresponding SU(2) theory.

We have chosen to concentrate on the non-compact model. To be exact, only the kinetic term for the gauge field is written in the non-compact form; for the kinetic term of the scalar field we use the compact formulation. This formalism has several advantages:

- It follows closer the continuum theory, so it should be easier to approach the continuum limit.
- Our results show that with relatively small volumes one gets quite close to the thermodynamic limit.
- The spurious U(1) monopoles, present in the compact formulation will not be present any more.
- The non-compact version is closer to the exact lattice form of the Landau-Ginzburg theory of superconductivity. The variable x of our model, defined in the text below, corresponds to κ^2 of the above theory. We recall that the phase transition is of first order for type I superconductors ($\kappa < \frac{1}{\sqrt{2}}$), corresponding to small Higgs mass; the converse holds for type II superconductors.

We have concentrated on the phase transition line. We have chosen to fix the Higgs mass to a fixed value (30 GeV), g to 1/3, m_W to 80.6 GeV and study the characteristics of the phase transition.

2 Reduction of the four-dimensional theory to three dimensions

The Lagrangian for the U(1) gauge-Higgs model in four dimensions is well known:

$$L_{4D} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + |D_\mu \varphi|^2 + m^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 \quad (1)$$

The action for this model at finite temperature is:

$$\begin{aligned} S[A_\mu(\tau, \mathbf{x}), \varphi(\tau, \mathbf{x})] \\ = \int_0^\beta d\tau \int d^3x \left[\frac{1}{4} F_{\mu\nu} F_{\mu\nu} \right. \\ \left. + |D_\mu \varphi|^2 + m^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2 \right], \quad (2) \end{aligned}$$

where $\beta = 1/T$.

If the action is expressed in terms of Fourier components, the mass terms are of the type:

$$[(2\pi nT)^2 + (\mathbf{k})^2] |A_\mu(n, \mathbf{k})|^2 \quad (3)$$

$$[(2\pi nT)^2 + (\mathbf{k})^2] |\varphi(n, \mathbf{k})|^2, \quad (4)$$

where $n = -\infty, \dots, \infty$.

At high temperatures T and energy scales less than $2\pi T$ the non-static modes $A_\mu(n \neq 0, \mathbf{k})$, $\varphi(n \neq 0, \mathbf{k})$ are thus suppressed by the factor $(2\pi nT)^2$ relative to the static $A_\mu(n = 0, \mathbf{k})$ and $\varphi(n = 0, \mathbf{k})$ modes. The method of dimensional reduction consists in integrating out the non-static modes in the action and deriving an effective action [2, 3].

An important remark is that the mass of the adjoint Higgs field is of order gT , which is large compared to $g^2 T$, the typical scale of the theory. Thus one can go on one step further and integrate it out using perturbation theory [5-7].

The effective action may then be written in the form:

$$\begin{aligned} S_{3D \text{ eff}}[A_i(\mathbf{x}), \varphi_3(\mathbf{x})] = \int d^3x \left[\frac{1}{4} F_{ij} F_{ij} + |D_i \varphi_3|^2 \right. \\ \left. + m_3^2 \varphi_3^* \varphi_3 + \lambda_3 (\varphi_3^* \varphi_3)^2 \right] \quad (5) \end{aligned}$$

The index 3 in (5) denotes the 3D character of the theory. The relations between the 4D and 3D parameters are (up to 2 loops):

$$g_3^2 = g^2(\mu)T, \quad (6)$$

$$\lambda_3 = T \left(\lambda(\mu) + \frac{2}{(4\pi)^2} g^4 \right) - \frac{g_3^4}{8\pi m_D}, \quad (7)$$

$$\begin{aligned} m_3^2(\mu_3) = \frac{1}{4} g_3^2 T + \frac{1}{3} \left(\lambda_3 + \frac{g_3^4}{8\pi m_D} \right) T \\ + \frac{g_3^2}{16\pi^2} \left(-\frac{8}{9} g_3^2 + \frac{2}{3} \left(\lambda_3 + \frac{g_3^4}{8\pi m_D} \right) \right) - \frac{1}{2} m_H^2 \\ + \frac{f_{2m}}{16\pi^2} \log \left(\frac{3T}{\mu_3} + c \right) - \frac{g_3^2 m_D}{4\pi} \end{aligned}$$

$$- \frac{g_3^4}{8\pi^2} \left(\log \frac{\mu_3}{2m_D} + \frac{1}{2} \right), \quad (8)$$

$$m_D^2 = \frac{1}{3} g^2(\mu) T^2. \quad (9)$$

We note that $f_{2m} = -4g_3^4 + 8\lambda_3 g_3^2 - 8\lambda_3^2$ and $c = -0.348725$ [5-7].

The couplings g_3^2 , λ_3 of the three-dimensional theory are renormalization group invariant because the theory is supernormalisable. The mass parameter m_3^2 contains a linear and a logarithmic divergence.

It is convenient to use the new set of parameters (g_3^2, x, y) rather than the set $(g_3^2, \lambda_3, m_3^2)$. x, y are defined by [8]:

$$x = \frac{\lambda_3}{g_3^2} \quad (10)$$

$$y = \frac{m_3^2(g_3^2)}{g_3^4} \quad (11)$$

It is evident that x is just proportional to the ratio of the squares of the scalar and vector masses; on the other hand, y is related to the temperature. The parameters x, y can be expressed in terms of the four-dimensional parameters as follows [10, 11]:

$$x = \frac{1}{2} \frac{m_H^2}{m_W^2} - \frac{\sqrt{3}g}{8\pi} \quad (12)$$

$$\begin{aligned} y = \frac{1}{4g^2} + \frac{1}{3g^2} \left(x + \frac{\sqrt{3}g}{8\pi} \right) \\ + \frac{1}{16\pi^2} \left(-\frac{8}{9} + \frac{2}{3} \left(x + \frac{\sqrt{3}g}{8\pi} \right) \right) - \frac{1}{4\pi\sqrt{3}g} \\ - \frac{1}{8\pi^2} \left(\log \frac{3\sqrt{3}}{2g} + c + \frac{1}{2} \right) - \frac{m_H^2}{2g^4 T^2} \\ + \frac{1}{16\pi^2} (-4 + 8x - 8x^2) \left(\log \frac{3}{g^2} + c \right) \quad (13) \end{aligned}$$

3 The lattice action

Discretizing the continuum action (5) we get:

$$\begin{aligned} S = \beta_g \sum_x \sum_{0 < i < j} F_{ij}^2 \\ + \beta_h \sum_x \sum_{0 < i} \left[\varphi^*(x) \varphi(x) - \varphi^*(x) U_i(x) \varphi(x + \hat{i}) \right] \\ + \sum_x \left[(1 - 2\beta_R - 3\beta_h) \varphi^*(x) \varphi(x) + \beta_R (\varphi^*(x) \varphi(x))^2 \right], \quad (14) \end{aligned}$$

where $F_{ij} = \Delta_i^f A_j(x) - \Delta_j^f A_i(x)$, $U_i(x) = e^{iA_i(x)}$.

Notice that we use the non-compact version for the gauge field as explained in the introduction. The naive continuum limit corresponds to the values: $\beta_g = \infty$, $\beta_h = \frac{1}{3}$, $\beta_R = 0$.

The lattice parameters and the (three-dimensional) continuum ones are related as follows [9]:

$$\beta_g = \frac{1}{ag_3^2} \quad (15)$$

$$\beta_R = \frac{x\beta_h^2}{4\beta_g} \quad (16)$$

$$2\beta_g^2 \left(\frac{1}{\beta_h} - 3 - \frac{2\beta_R}{\beta_h} \right) = y - (2 + 4x) \frac{\Sigma\beta_g}{4\pi} - \frac{1}{16\pi^2} [(-4 + 8x - 8x^2) \times (\log 6\beta_g + 0.09) - 1.1 + 4.6x]. \quad (17)$$

We note that $\Sigma = 3.176$ at the scale $\mu_3 = g_3^2$.

4 The algorithm

We used the Metropolis algorithm for the updating of both the gauge and the Higgs field. It is known that the scalar fields have much longer autocorrelation times than the gauge fields. Thus, special care must be taken to increase the efficiency of the updating for the Higgs field. We made the following additions to the Metropolis updating procedure [8]:

a) Global radial update: We update the radial part of the Higgs field by multiplying it by the same factor at all sites: $R(\mathbf{x}) \rightarrow e^\xi R(\mathbf{x})$, where $\xi \in [-\varepsilon, \varepsilon]$ is randomly chosen. The quantity ε is adjusted such that the acceptance rate is kept between 0.6 and 0.7. The probability for the updating is $P(\xi) = \min\{1, \exp(2V\xi - \Delta S(\xi))\}$ where $\Delta S(\xi)$ is the change in action, while the $2V\xi$ term comes from the change in the measure.

b) Higgs field overrelaxation: We write the Higgs potential at \mathbf{x} in the form:

$$V(\varphi(\mathbf{x})) = -\mathbf{a} \cdot \mathbf{F} + R^2(\mathbf{x}) + \beta_R (R^2(\mathbf{x}) - 1)^2 \quad (18)$$

where

$$\mathbf{a} \equiv \begin{pmatrix} R(\mathbf{x}) \cos \chi(\mathbf{x}) \\ R(\mathbf{x}) \sin \chi(\mathbf{x}) \end{pmatrix},$$

$$\mathbf{F} \equiv \begin{pmatrix} \beta_h \sum_i R(\mathbf{x} + \hat{i}) \cos(\chi(\mathbf{x} + \hat{i}) + \theta(\mathbf{x})) \\ \beta_h \sum_i R(\mathbf{x} + \hat{i}) \sin(\chi(\mathbf{x} + \hat{i}) + \theta(\mathbf{x})) \end{pmatrix}.$$

We can perform the change of variables: $(\mathbf{a}, \mathbf{F}) \rightarrow (X, F, \mathbf{Y})$, where

$$F \equiv |\mathbf{F}|, \quad \mathbf{f} \equiv \frac{\mathbf{F}}{\sqrt{F_1^2 + F_2^2}}, \quad X \equiv \mathbf{a} \cdot \mathbf{f}, \quad \mathbf{Y} \equiv \mathbf{a} - X\mathbf{f}. \quad (19)$$

The potential may be rewritten in terms of the new variables:

$$\bar{V}(X, F, \mathbf{Y}) = -XF + (1 + 2\beta_R(\mathbf{Y}^2 - 1))X^2 + \mathbf{Y}^2(1 - 2\beta_R) + \beta_R(X^4 + \mathbf{Y}^4). \quad (20)$$

The updating of \mathbf{Y} is done simply by the reflection:

$$\mathbf{Y} \rightarrow \mathbf{Y}' = -\mathbf{Y}. \quad (21)$$

The updating of X is performed by solving the equation:

$$\left(\frac{\partial \bar{V}(X', F', \mathbf{Y}')}{\partial X'} \right)^{-1} \exp(-\bar{V}(X', F', \mathbf{Y}')) = \left(\frac{\partial \bar{V}(X, F, \mathbf{Y})}{\partial X} \right)^{-1} \exp(-\bar{V}(X, F, \mathbf{Y})). \quad (22)$$

The change $X \rightarrow X'$ is accepted with probability: $P(X') = \min\{P_0, 1\}$, where $P_0 \equiv \frac{\partial \bar{V}(X, F, \mathbf{Y})}{\partial X} / \frac{\partial \bar{V}(X', F', \mathbf{Y}')}{\partial X'}$.

5 Results

For our Monte-Carlo simulations we used cubic lattices with volumes $V = 12^3, 16^3, 24^3$. For each volume we performed 60 000 to 110 000 thermalization sweeps and 70 000 to 120 000 measurements. We have set the value of x equal to 0.0463. According to the relation (12), using $m_W = 80.6$ GeV and $g = \frac{1}{3}$, this value of x corresponds to a Higgs field mass $m_H = 30$ GeV. We used two values for β_g , namely $\beta_g = 4$ and $\beta_g = 8$. For each value of β_h we use the relation (16) to determine the corresponding β_R . This value of x has been used in references [10, 11] in the study of the compact U(1) model, so we use the same value to facilitate comparison. The two models should be close for large values of β_g , where the compact formulation probably approaches the non-compact one. The phase transition is expected to be of first order, since the mass of the scalar field is safely low.

We used four quantities to locate the phase transition points:

1. The distribution $N(E_{link})$ of E_{link} .
2. The susceptibility of $E_{link} \equiv \frac{1}{3V} \sum_{x,i} \Omega^*(x) U_i(x) \Omega(x+i)$ (we have set $\varphi(x) \equiv R(x)e^{i\chi(x)} \equiv R(x)\Omega(x)$):

$$S(E_{link}) \equiv V(\langle (E_{link})^2 \rangle - \langle E_{link} \rangle^2).$$

3. The susceptibility of $R^2 \equiv \frac{1}{V} \sum_x R^2(x)$:

$$S(R^2) \equiv V(\langle (R^2)^2 \rangle - \langle R^2 \rangle^2).$$

4. The Binder cumulant of E_{link} :

$$C(E_{link}) = 1 - \frac{\langle (E_{link})^4 \rangle}{3\langle (E_{link})^2 \rangle^2}.$$

We have searched for the (pseudocritical) β_h values yielding (a) equal heights of the two peaks of the distribution $N(E_{link})$, (b) the maxima of the quantities $S(E_{link})$, $S(R^2)$ and (c) the minima of the cumulant $C(E_{link})$. Of course, the values $\beta_h^*(A, V)$ found using each of the above four quantities, depend on the specific quantity (denoted by A) which has been employed, as well as on the volume V . While searching, we have made use of the Ferrenberg-Swendsen reweighting technique [4] to find the pseudocritical β_h for the volume 24^3 .

In Fig. 1 we show an example of the distribution of E_{link} in a 16^3 lattice for $\beta_g = 8$ and three values of β_h :

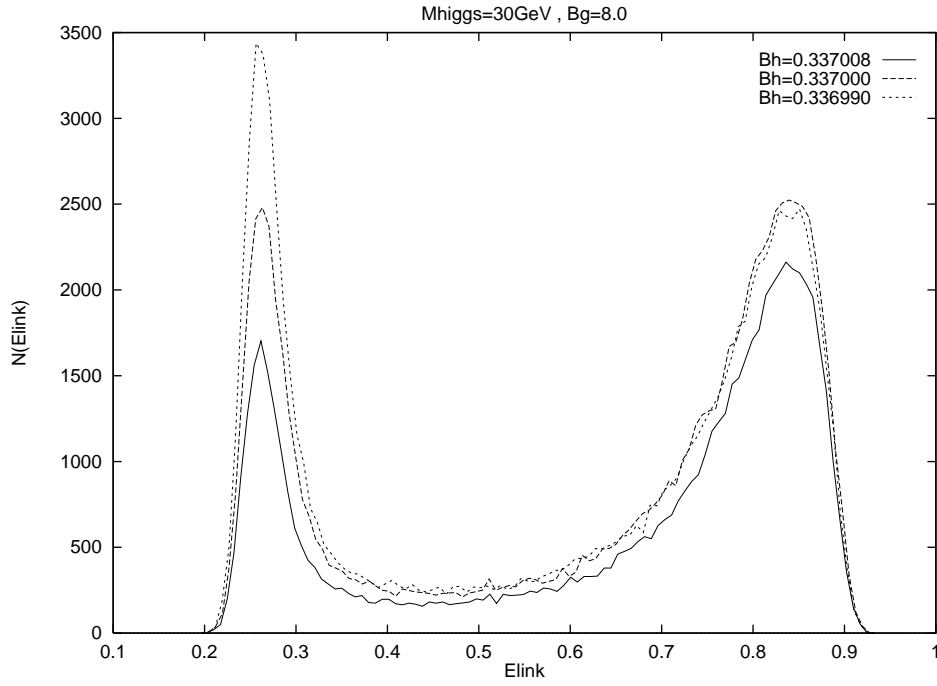


Fig. 1. Distribution of E_{link}

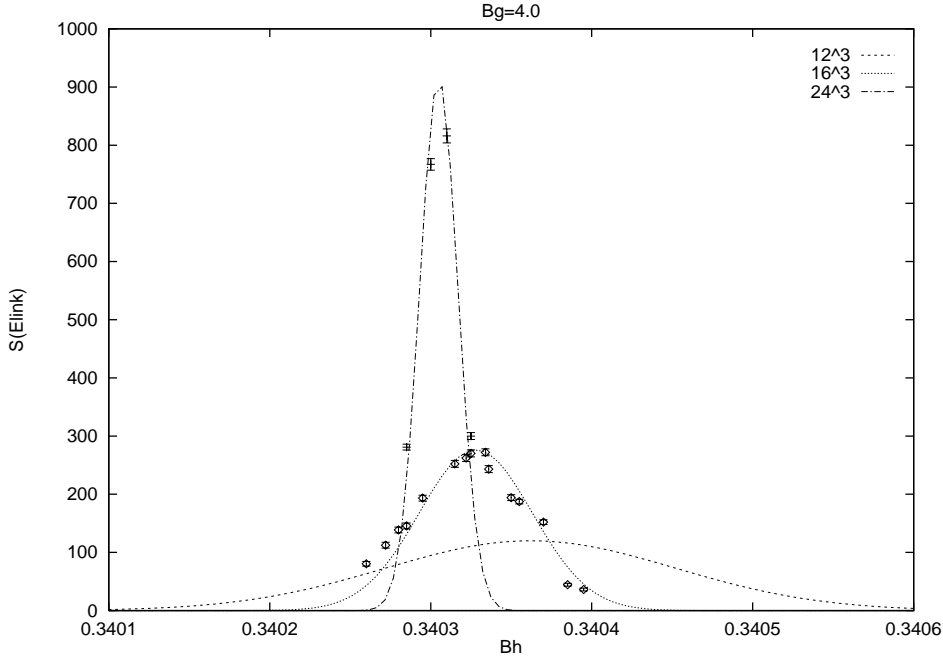
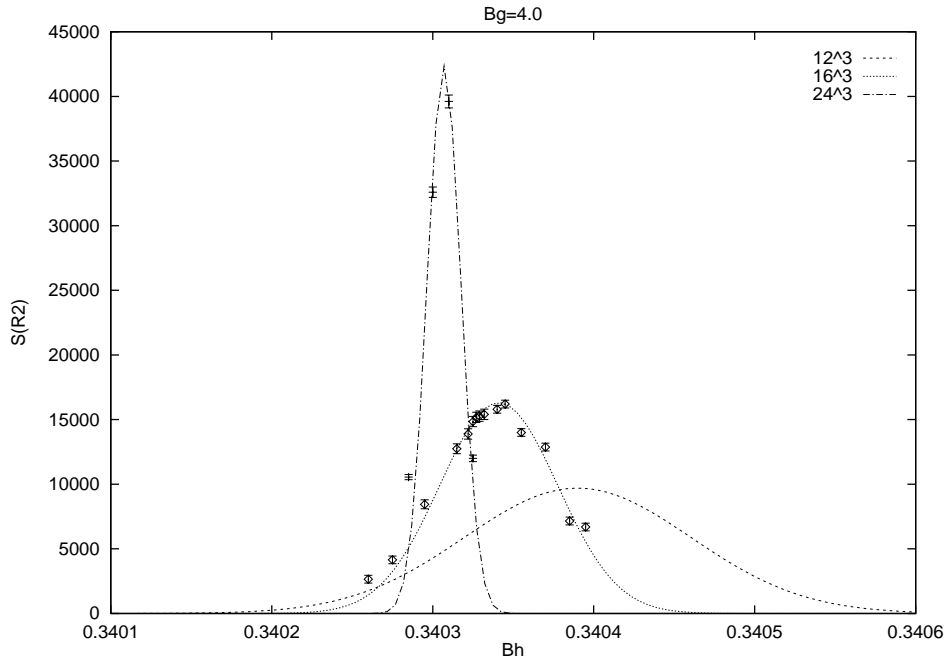
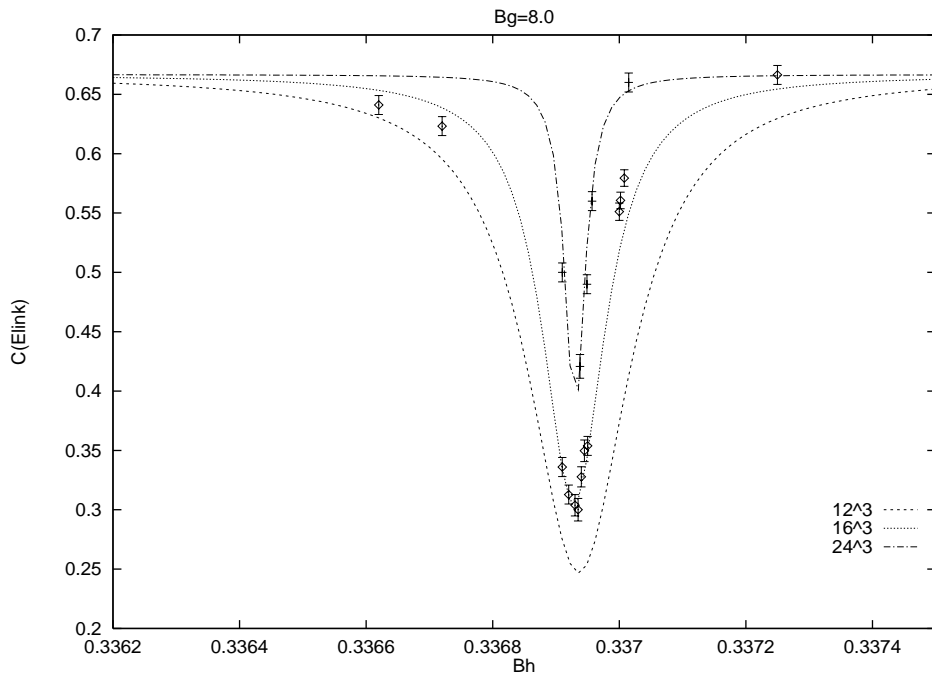


Fig. 2. Susceptibility of E_{link}

the pseudocritical one (0.337000), one somewhat smaller and one somewhat larger than this value. This is just to illustrate the way in which the equal height criterion for the critical point has worked. It is clear from the figure that the pseudocritical β_h yields two maxima of equal height in the distribution. For the “small” β_h the peak in the region of small values of E_{link} is more pronounced, while for the “large” β_h it is the other way around. The picture of the two well separated peaks at criticality is the signature of a first order phase transition; the two peaks correspond to the two coexisting metastable states.

In Figs. 2 and 3 we depict the behaviour of the susceptibilities $S(E_{link})$ and $S(R2)$ for $\beta_g = 4$ versus β_h for three lattice volumes. We have fitted curves through the data and show them in the figures; for the 16^3 and 24^3 lattice volumes we also give the actual measurements. It is evident that the curves represent the data quite nicely. To calculate the error bars we first found the integrated autocorrelation times $\tau_{int}(A)$ for the relevant quantities A and constructed samples of data separated by a number of steps greater than $\tau_{int}(A)$. The errors have been calculated by the Jackknife method, using the samples constructed according to the procedure just described. We observe that


Fig. 3. Susceptibility of R_2

Fig. 4. Binder cumulant of E_{link}

the peak values for the susceptibilities increase almost linearly with the volume in both cases, which is evidence for a first order phase transition. In Fig. 4 we depict the behaviour of the Binder cumulant $C(E_{link})$ at $\beta_g = 8$ for three lattice sizes. We again show the real measurements for 16^3 and 24^3 only and just give the fitted curves for 12^3 . The error bars have been calculated by the Jackknife method [13], in the same way as in the case of the susceptibilities. The volume dependence of the cumulants is again characteristic of a first order phase transition.

The use of finite lattices is the reason why the $\beta_h^*(A, V)$ values that we have found employing the various criteria are slightly different. Thus, one should extrapolate these

values to infinite volume. We have adopted the ansatz:

$$\beta_h^*(A, V) = \beta_h^{cr}(\infty) + \frac{c(A)}{V},$$

The constant $c(A)$ is expected to depend on the quantity A , while the extrapolated value $\beta_h^{cr}(\infty)$ should not depend on A ; that is, the infinite volume extrapolation for the critical point should not depend on the quantity used.

Figures 5 and 6 deal with the extrapolation to infinite volume for $\beta_g = 4$ and $\beta_g = 8$ respectively. They contain the data for the pseudocritical $\beta_h^*(A, V)$ values obtained from the various quantities A versus the inverse lattice volume, along with the linear fits to the data. The error

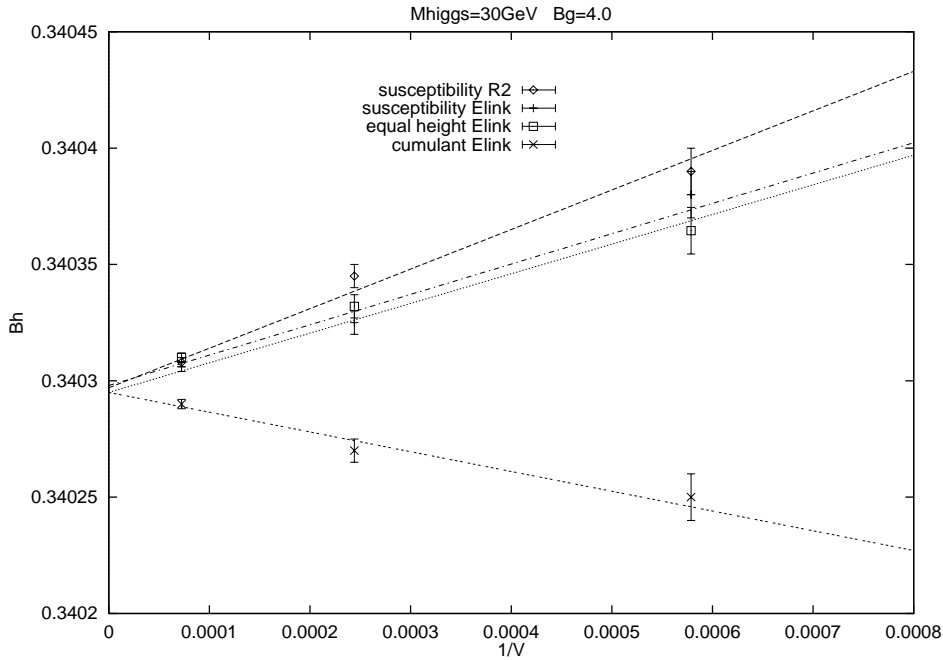


Fig. 5. Extrapolation for $\beta_g = 4$

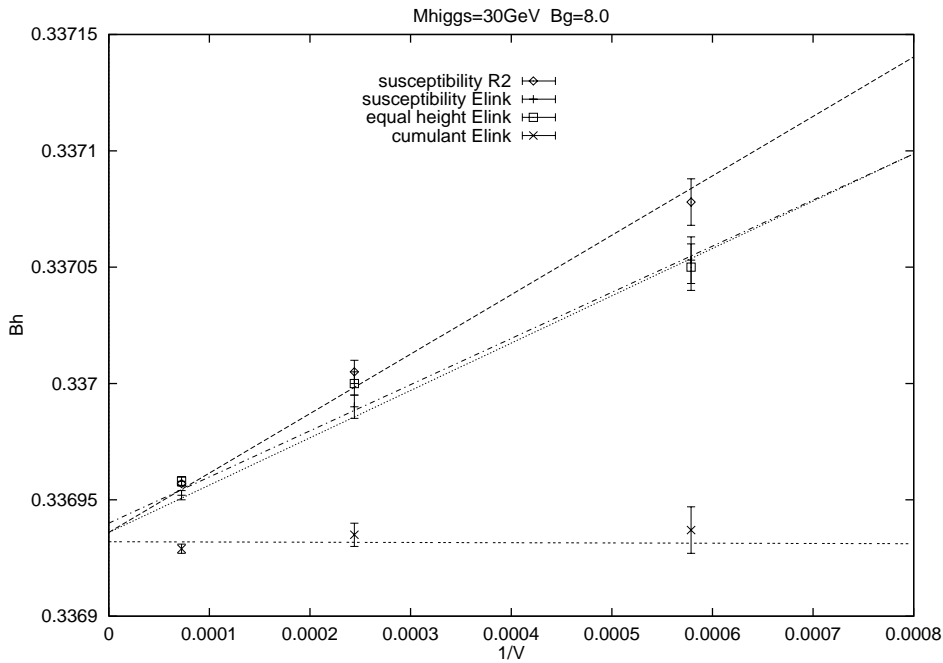
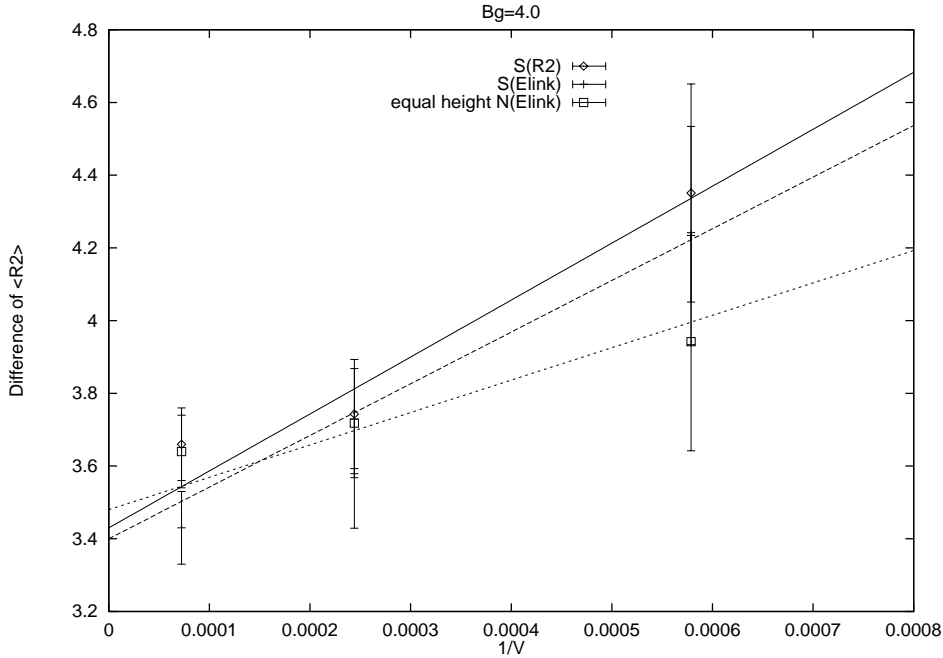
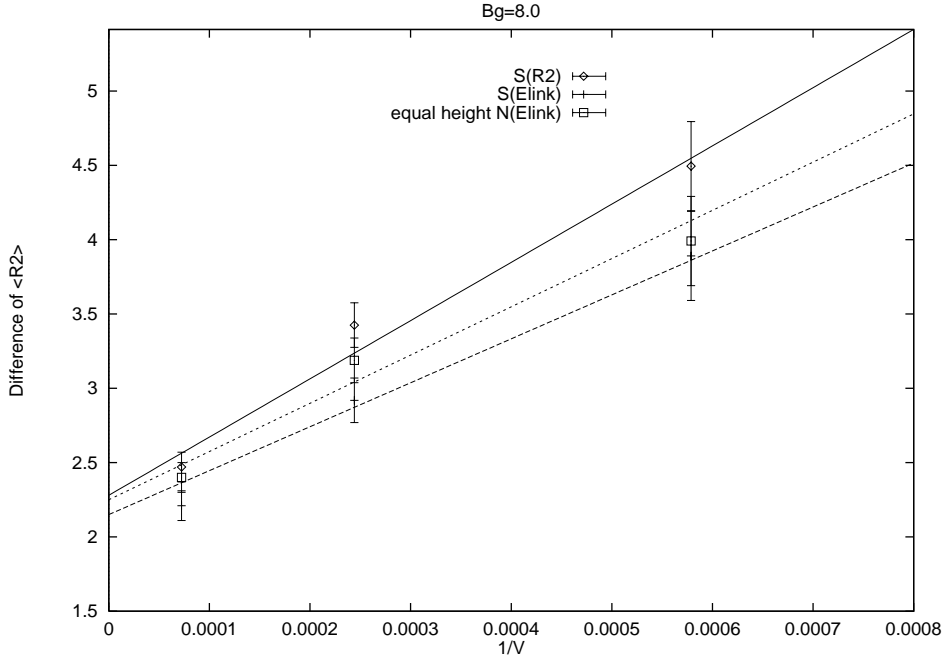


Fig. 6. Extrapolation for $\beta_g = 8$

bars in β_h have been found from the statistical error of the values of the quantities A at the critical point. We note in passing that, at finite volumes, the smallest pseudocritical values are given by the cumulant of E_{link} ; then follow, in ascending order, the values given from the equal height, the susceptibility of E_{link} and the susceptibility of $R2$. This holds for both values of β_g . One may also observe that the infinite volume extrapolation is almost independent from the specific quantity used: the differences at the point $\frac{1}{V} = 0$ between the various extrapolated critical values are less than 10^{-5} . To be specific, the critical values lie in the interval $(0.340295, 0.340298)$ for $\beta_g = 4$ and in $(0.336932, 0.336940)$ for $\beta_g = 8$. In reference [10], treat-

ing the compact U(1) model, the best pseudocritical value corresponding to our results has been obtained for $\beta_g = 8$ for a 32^3 lattice. From the relevant figure one may read out a pseudocritical value about 0.3370, a result consistent with ours. These results suggest that the non-compact formulation allows one to obtain similar results to the ones of the compact formulation in a quite economical way.

The next quantity we are going to deal with is the critical temperature. It may be determined by noting that, for each β_g , the quantity $\beta_h^{cr}(\infty)$ yields y_{cr} through (16, 17); then (13) gives T_{cr} . The results are to be found in Table 1. We mention that in reference [8] the critical temperature for the SU(2) model at $\beta_g = 8$ and $m_H = 35$ GeV has


Fig. 7. Extrapolation for $\Delta\langle R2\rangle$

Fig. 8. Extrapolation for $\Delta\langle R2\rangle$
Table 1.

β_g	T_{cr}	L/T_{cr}^4	$\langle\varphi^*\varphi\rangle/T_{cr}$
4	131.50(3)	0.0135(4)	0.255(6)
8	131.18(14)	0.0172(6)	0.308(10)
pert.	132.64	0.0150	0.285

been found 94.181 GeV. No lattice result is reported for this quantity in the paper [11] on compact U(1), but there is the result 148.83 GeV at $m_H = 35$ GeV from the perturbative effective potential. We will say more about this later on, but we remark at this point that this value is quite close to ours.

Having found the critical temperature we estimated the latent heat, that is the energy released in the transition. We have used the formula [8, 9]:

$$\frac{L}{T_{cr}^4} = \frac{1}{2} \frac{M_H^2}{T_{cr}^3} g_3^2 \beta_h^{cr} \beta_g \Delta\langle R2\rangle. \quad (23)$$

We note that $g_3^2 = g^2 T_{cr}$. The quantity $\Delta\langle R2\rangle$ is the difference of the $R2$ expectation values between the phases. We have measured the values of $\Delta\langle R2\rangle$ from the $R2$ distributions for each lattice volume at the three different pseudocritical values of $\beta_h^*(A, V)$ [12] the quantities A being the susceptibilities of E_{link} and $R2$ and the equal height signal of $N(E_{link})$. Figures 7 and 8 show these sets of three

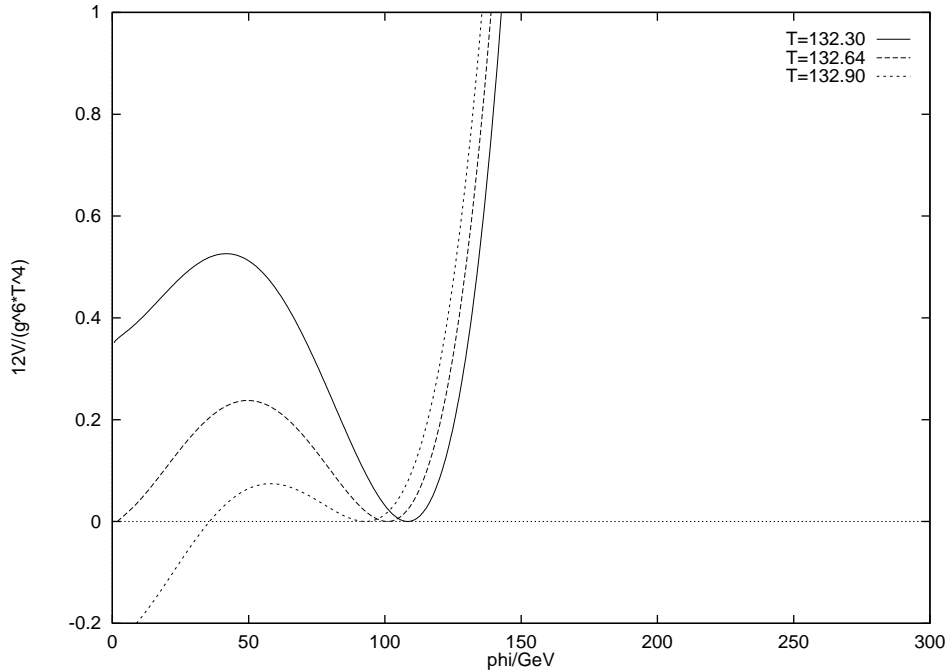


Fig. 9. 2-loop effective potential. We use $\phi = \varphi\sqrt{T}$

measurements versus the inverse volume of the lattice for the cases $\beta_g = 4$ and $\beta_g = 8$. The error bars are due to the uncertainty of each pseudocritical value, as well as to statistical dispersion; they turn out to be rather big, especially for the smallest volume. The final values of $\Delta\langle R2 \rangle$, which have been used for the calculation of $\frac{L}{T_{cr}^4}$ have been found from a linear fit of these data with $\frac{1}{V}$. The convergence of the three straight lines to a common value in the limit of infinite volume is fairly good. The results for the quantity $\frac{L}{T_{cr}^4}$ can be found in Table 1. The corresponding results for the SU(2) case at $m_H = 35$ GeV have been reported to be 0.180 ± 0.001 in [12] and 0.256 ± 0.008 in [8]. The former result has been obtained for $\beta_g = 12$, while for the latter an extrapolation to $\beta_g = \infty$ has been done. We observe that our values of this quantity for U(1) are less than the one tenth of the values for SU(2). This permits one to be sure that the U(1) part of the Standard Model gauge group plays only a secondary role in the scenario of the Electroweak Phase Transition. The relative factor of ten is so big, that this conclusion cannot be spoiled by the rather large errors.

For the (3-dimensional) lattice quantity $\langle \varphi^* \varphi \rangle / T_{cr}$, also appearing in Table 1, it is important that one subtracts the “infinities” from the lattice results. The relevant formula reads [9]:

$$\langle \varphi^* \varphi \rangle = \frac{1}{2} \beta_h \beta_g g_3^2 \langle R2 \rangle - \frac{g_3^2 \beta_g \Sigma}{4\pi} - \frac{g_3^2}{8\pi^2} \left[\log 6\beta_g + \zeta + \frac{\Sigma^2}{4} - \delta \right],$$

where $\Sigma = 3.176$, $\zeta = 0.09$ and $\delta = 1.94$.

One may use the U(1) effective potential [5] to determine the critical temperature T_{cr} , as well as the quantity $\langle \varphi^* \varphi \rangle / T_{cr}$ and $\frac{L}{T_{cr}^4}$ and compare with the corresponding

quantities from the lattice. (We note that the critical temperature is defined in perturbation theory by the equality of the two minima of the potential.) The perturbative predictions are also displayed in Table 1.

Figure 9 depicts the two-loop effective potential versus the four-dimensional scalar field (a) for the critical temperature $T_{cr, \text{pert}}$, and (b) for two other neighbouring temperatures, one corresponding to the symmetric phase and the other to the broken phase.

In principle one should perform the extrapolation to large values of β_g . However one is not sure about the exact β_g dependence of the various quantities, so we postpone this until we get results for even bigger β_g .

We observe that the lattice T_{cr} is smaller than the prediction from perturbation theory and decreases with β_g (in agreement with the SU(2) results for small m_H [8]). The other two quantities that we measured, namely $\frac{L}{T_{cr}^4}$ and $\langle \varphi^* \varphi \rangle / T_{cr}$, increase with β_g and their values are compatible with the perturbative ones.

Acknowledgements. K.F. thanks M.Laine for a crucial observation concerning equation (17). This work has been supported in part by the PENED 95 Program, No 1170 of the Greek General Secretariat of Research and Technology. Thanks are also due to the Computer Center of the NTUA and the Silicon Graphics Hellas Ltd for the computer time allocated.

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